Advanced Topics on Beam Bending

ME 322: Mechanics of Solids II
Lecture 8

Topics for Today

- Bending of Nonprismatic Beam
- Composite Beam Bending
- Bending under inclined loads
- Plastic Bending

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Bending of Nonprismatic Beam

- Normally, section modulus is constant

$$\sigma_1 = -\frac{Mc_1}{I} = -\frac{M}{S_1}; S_1 = \frac{I}{c_1}$$

$$\sigma_2 = -\frac{Mc_2}{I} = -\frac{M}{S_2}; S_2 = \frac{I}{c_2}$$

- However, in nonprismatic beam, section modulus varies with the beam geometry

Example (Timoshenko & Gere 5-10)

Strain in Composite Beam

$$\varepsilon_x(y) = -\frac{y}{R} = -\kappa y$$

- Same representation still holds
- However, neutral axis is a little more complicated to find

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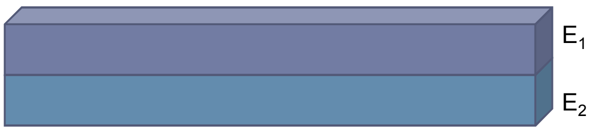
Neutral axis for a composite beam



$$\int_1 \sigma_{x1} dA + \int_2 \sigma_{x2} dA = 0$$

$$-\int_1 E_1 \kappa y dA - \int_2 E_2 \kappa y dA = 0$$

$$E_1 \int_1 y dA - E_2 \int_2 y dA = 0$$

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Moment Curvature Relationship

$$M = -\int_1 \sigma_{x1} y dA - \int_2 \sigma_{x2} y dA$$
$$= -\kappa E_1 \int_1 y^2 dA - \kappa E_2 \int_2 y^2 dA$$
$$= \kappa (E_1 I_1 + E_2 I_2)$$

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Normal Stress in Composite Beam Bending

$$\sigma_{x1} = -\frac{MyE_1}{E_1 I_1 + E_2 I_2}$$
$$\sigma_{x2} = -\frac{MyE_2}{E_1 I_1 + E_2 I_2}$$

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Example

Bending of Doubly Symmetric Beams with Inclined Loads

- Using superposition:

$$\sigma_x = \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

- Neutral axis

$$\tan \beta = \frac{y}{z} = \frac{M_y I_z}{M_z I_y}$$

Elastoplastic Bending

- Similar to deformation of other forms, bending can also lead to plastic deformation
- Bending moment just as the maximum stress reaches yield is

$$M_y = \frac{\sigma_y I}{c} = \sigma_y S$$

- As the bending continues, more of the cross section will yield
  - Maximum bending moment a beam can take is called the *plastic moment*

Plastic Moment

- To find it, we first need to find the neutral axis under fully plastic bending
- What is the neutral axis for fully plastic bending?
  - It may not be the same as that of a linearly elastic bending
- The plastic moment is

$$M_p = -\int_A \sigma y dA = -\int_{A_1} (-\sigma_y) y dA - \int_{A_2} \sigma_y y dA$$

$$= \sigma_y (\bar{y}_1 A_1) - \sigma_y (\bar{y}_2 A_2) = \frac{\sigma_y A(\bar{y}_1 + \bar{y}_2)}{2}$$

$$M_p = \sigma_y Z$$

Shape Factor (f)

- Measures the reserve strength of the beam after yielding begins

$$f = \frac{M_p}{M_y} = \frac{Z}{S}$$

Example: Beam of rectangular cross section

Topic for Today

- Beam Deflection
  - Direct Integration
  - Energy Method